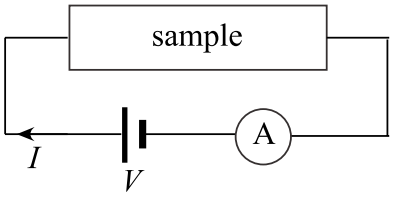

Figure 1. Battery and amperemeter connected to the sample.

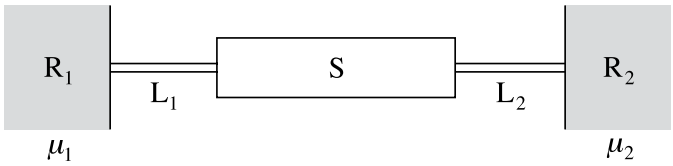

Figure 2. The reservoirs R_1 and R_1 connected to the sample S through the leads L_1 and L_2.

which is applicable to various phenomena [2]. Among them, the most important is the one on electron conduction. In the same year Landauer proposed a unique theory of electron conduction. Although these theories have completely different appearances, in some cases they give an identical result. In the following I will review the essential aspects of these two theories. Landauer's paper was published after Kubo's paper, but it is instructive to review the Landauer theory first.

2.1. The Landauer theory

When we measure the conductance of a sample, we connect a battery and an amperemeter as in figure 1. If the resistances of the parts other than the sample are negligible, the conductance of the sample is given by $G \equiv I/V$, namely, current I through the sample divided by the voltage V of the battery. It is rather difficult to treat a realistic model of a battery, and Landauer introduced electron reservoirs which play the role of the battery. As in figure 2, reservoirs R_1 and R_1 are connected to the sample S through leads L_1 and L_2. Let μ_1 and μ_2 be the chemical potentials of electrons in R_1 and R_2, respectively ($\mu_1 > \mu_2$). Then the electric potential difference between R_1 and R_2 is given by

$$V = \frac{\mu_1 - \mu_2}{e}, \tag{2.1}$$

where e is the absolute value of the electronic charge.

In the following, we will confine ourselves to the case of one-dimensional systems at zero temperature. Here a one-dimensional system means the leads and the sample are one-dimensional. The following postulates are essential in Landauer's theory.

(1) The leads are ideal and the electrons are not scattered in them.
(2) No inelastic scattering occurs in the sample.
(3) In the lead L_1, all the right going electronic state of the energy between μ_1 and μ_2 are occupied.
(4) The reservoirs are large enough so that the chemical potentials do not change if electrons are transferred from one to the other.

(5) When an electron is injected into L_1 from R_1, the hole left in R_1 immediately relaxes to μ_1, and an electron injected into R_2 from L_2 immediately relaxes to μ_2.
(6) The wavefunctions of the electrons injected into L_1 are incoherent.

For the moment we consider the case when the transmission probability of electrons through the sample is unity. The electrons in the states below μ_2 do not contribute to the current, for all the right going as well as left going states are occupied in the leads. We denote the energy of an electron of wave number k in the leads by $\varepsilon(k)$, and we let k_i $(i = 1, 2)$ be such wave numbers that $\varepsilon(k_i) = \mu_i$ $(k_i > 0)$. Then, from the above postulates, we easily find that the current from R_2 to R_1 is given by

$$I = 2e \int_{k_2}^{k_1} \frac{\mathrm{d}\varepsilon(k)}{\hbar \mathrm{d}k} \frac{\mathrm{d}k}{2\pi} = \frac{e}{\pi\hbar} \int_{\mu_2}^{\mu_1} \mathrm{d}\varepsilon = \frac{e(\mu_1 - \mu_2)}{\pi\hbar}, \tag{2.2}$$

where $\mathrm{d}\varepsilon(k)/(\hbar \mathrm{d}k)$ is the group velocity of an electron in the leads. Therefore, using equation (2.1) we find that the conductance is given by

$$G \equiv \frac{I}{V} = \frac{e^2}{\pi\hbar} = \frac{2e^2}{h}, \tag{2.3}$$

with $h = 2\pi\hbar$. The right-hand side is independent of the details of the structures of the system and is called *quantized conductance*.

If the transmission probability $\mathcal{T}$ through the sample is smaller than unity, the current is reduced by a factor $\mathcal{T}$ and the conductance becomes

$$G = \frac{2e^2}{h}\mathcal{T}. \tag{2.4}$$

One may suspect that the resistance $1/G$ vanishes when the transmission probability is unity, for no scattering occurs in the leads and the sample. This finite resistance is interpreted by Imry as the *contact resistance* between the reservoirs and the leads [5].

So far we have considered the case when the temperature is zero and the transmission probability is almost independent of the energy of the incident electron. For resonant transmission, which will be discussed in the last part of section 7, the transmission probability $\mathcal{T}(\varepsilon)$ is a rapidly varying function of the electron energy ε, and the formula (2.4) has to be modified.

At finite temperatures, the current from R_2 to R_1 and that from R_1 to R_2 carried by electrons of energy ε are proportional to $f_0(\varepsilon - \mu_1)(1 - f_0(\varepsilon - \mu_2))$ and $f_0(\varepsilon - \mu_2)(1 - f_0(\varepsilon - \mu_1))$, respectively, where

$$f_0(\varepsilon) \equiv \frac{1}{\mathrm{e}^{\beta\varepsilon} + 1}, \tag{2.5}$$

with $\beta \equiv 1/(k_\mathrm{B}T)$. Therefore, equation (2.2) should be modified as

$$I = \frac{e}{\pi\hbar} \int_0^\infty \{f_0(\varepsilon - \mu_1) - f_0(\varepsilon - \mu_2)\}\mathcal{T}(\varepsilon)\,\mathrm{d}\varepsilon. \tag{2.6}$$

Then the conductance is given by

$$G = \lim_{\mu_1, \mu_2 \to \mu} \frac{eI}{\mu_1 - \mu_2} = \frac{e^2}{\pi\hbar} \int_0^\infty \left(-\frac{\partial f_\mathrm{F}(\varepsilon)}{\partial\varepsilon}\right) \mathcal{T}(\varepsilon)\,\mathrm{d}\varepsilon, \tag{2.7}$$

where $f_\mathrm{F}(\varepsilon)$ is the Fermi distribution function:

$$f_\mathrm{F}(\varepsilon) \equiv \frac{1}{\mathrm{e}^{\beta(\varepsilon-\mu)} + 1}. \tag{2.8}$$